

# Analysis of Method Comparison Studies

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MEGA Center, SPH, University of Melbourne

## Comparing two methods with one measurement on each

Friday 15 February

### Bendix Carstensen

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15 February 2008  
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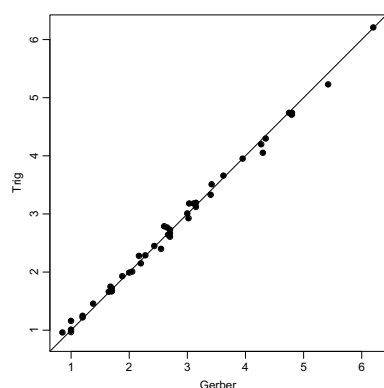
(Comp-simple)

## Comparing measurement methods

General questions:

- ▶ Are results systematically different?
- ▶ Can one method safely be replaced by another?
- ▶ What is the size of measurement errors?
- ▶ Different centres use different methods of measurement: How can we convert from one method to another?

## Two methods for measuring fat content in human milk:



The relationship looks like:

$$y_1 = a + by_2$$

## Two methods — one measurement by each

How large is the difference between a measurement with method 1 and one with method 2 on a (randomly chosen) person?

$$D_i = y_{1i} - y_{2i}, \quad \bar{D}, \quad \text{s.d.}(D)$$

“Limits of agreement:”

$$\bar{D} \pm 2 \times \text{s.d.}(D)$$

95% prediction interval for the difference between a measurement by method 1 and one by method 2.  
[?, ?]

Comparing two methods with one measurement on each

3/ 71

## Limits of agreement: Interpretation

- ▶ If a new patient is measured **once** with each of the two methods, the difference between the two values will with 95% probability be within the limits of agreement.
- ▶ This is a **prediction** interval for a (future) difference.
- ▶ Requires a **clinical** input:  
Are the limits of agreement sufficiently narrow to make the use of either of the methods clinically acceptable?
- ▶ Is it relevant to test if the mean is 0?

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4/ 71

## Limits of agreement: Test?

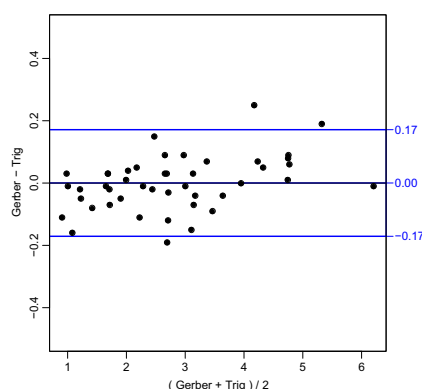
Testing whether the difference is 0 is a bad idea:

- ▶ If the study is sufficiently small this will be accepted even if the difference is important.
- ▶ If the study is sufficiently large this will be rejected even if the difference is clinically irrelevant.
- ▶ It is an **equivalence** problem:  
**Clinical input is required!**

Comparing two methods with one measurement on each

5/ 71

## Limits of agreement:



Plot differences ( $D_i$ ) versus averages ( $A_i$ ).

Comparing two methods with one measurement on each

2/ 71

Comparing two methods with one measurement on each

6/ 71

## Model in “Limits of agreement”

Methods  $m = 1, \dots, M$ , applied to  $i = 1, \dots, I$  individuals:

$$y_{mi} = \alpha_m + \mu_i + e_{mi}$$
$$e_{mi} \sim \mathcal{N}(0, \sigma_m^2) \quad \text{measurement error}$$

- ▶ Two-way analysis of variance model, with unequal variances in columns.
- ▶ Different variances are not identifiable without replicate measurements for  $M = 2$  because the variances cannot be separated.

Models

7/ 71

## Limits of agreement:

Unequal variances induce correlation between  $D_i$  and  $A_i$ :

$$\text{cov}(D_i, A_i) = \frac{1}{2}(\sigma_x^2 - \sigma_y^2) \neq 0 \quad \text{if } \sigma_x \neq \sigma_y$$

In correlation terms:

$$\rho(D, A) = \frac{1}{2} \frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2}$$

i.e. the correlation depends on whether the difference between the variances is large relative to the sizes of the two.

Models

8/ 71

## Limits of agreement:

Usually interpreted as the likely difference between two future measurements, one with each method:

$$\widehat{y_2 - y_1} = \hat{D} = \alpha_2 - \alpha_1 \pm 1.96 \text{ s.d.}(D)$$

But it can of course also be converted to a prediction interval for  $y_2$  given  $y_1$ :

$$\hat{y}_2 | y_1 = \alpha_2 - \alpha_1 + y_1 \pm 1.96 \text{ s.d.}(D)$$

Models

9/ 71

## Introduction to computing

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(Intro-comp)

## Course structure

The course is both theoretical and practical, i.e. the aim is to convey a basic understanding of the problems in method comparison studies, but also to convey practical skills in handling the statistical analysis.

- ▶ **R** for data manipulation and graphics.
- ▶ WinBUGS for estimation in non-linear variance component models.

Introduction to computing

10/ 71

## Software considerations

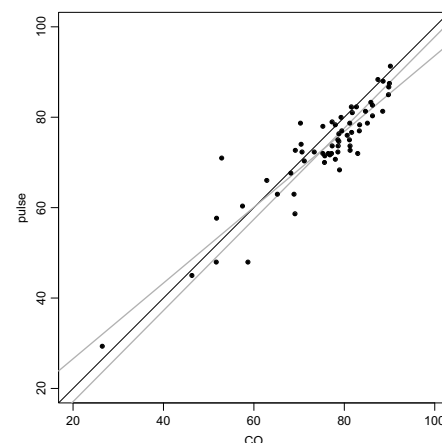
- ▶ **R**, SAS and Stata all have interfaces to WinBUGS.
- ▶ But **R** have more flexible graphical facilities.
- ▶ The MethComp package is written for **R**.

Therefore we use **R** in this course.

Introduction to computing

11/ 71

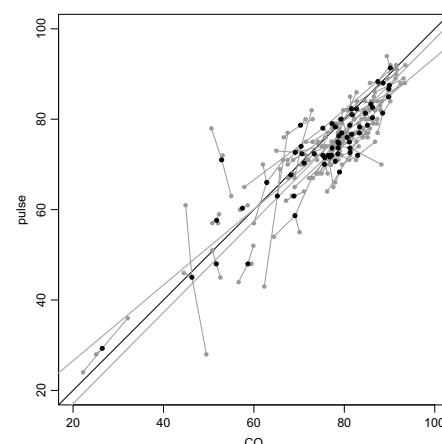
## Oximetry data



Introduction to computing

12/ 71

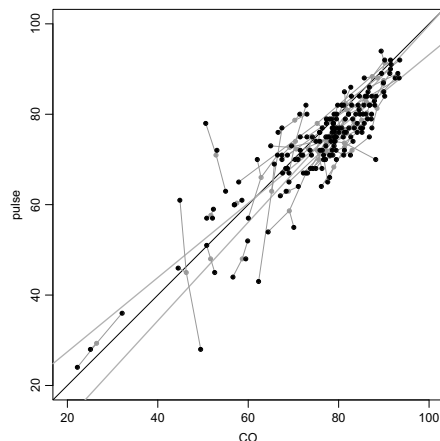
## Oximetry data



Introduction to computing

13/ 71

## Oximetry data



Linked  
replicates.

Introduction to computing

14/ 71

## Requirements

- ▶ **R** for data manipulation and graphics:
- ▶ Tinn-R convenience editor with syntax highlighting for **R**.
- ▶ nlme-package for variance component models — constant bias.
- ▶ WinBUGS for fitting models with linear bias (non-linear variance component models, over-parametrized).

All of it works from within **R**.

Introduction to computing

18/ 71

## How it works

Example data sets are included in the MethComp package. Contains the following variables.

meth — method  
item — item, person, individual, sample  
repl — replicate (if present)  
y — the actual measurement

— or rather *should* in order for the functions in MethComp to work.

Introduction to computing

15/ 71

## Functions in the MethComp package

5 broad categories of functions in MethComp:

- ▶ Graphical — just exploring data.
- ▶ Data manipulation — reshaping and changing.
- ▶ Simulation — generating datasets.
- ▶ Analysis function — fitting models to data.
- ▶ Reporting functions — displaying the results from analyses.

Introduction to computing

19/ 71

## How it looks

```
> subset(ox,item<3)
  meth item repl   y
1    CO    1   1  78.0
2    CO    1   2  76.4
3    CO    1   3  77.2
4    CO    2   1  68.7
5    CO    2   2  67.6
6    CO    2   3  68.3
184 pulse 1   1  71.0
185 pulse 1   2  72.0
186 pulse 1   3  73.0
187 pulse 2   1  68.0
188 pulse 2   2  67.0
189 pulse 2   3  68.0
```

```
> subset(to.wide(ox),item<3)
Note:
Replicate measurements are taken
  item repl id   CO pulse
1     1    1 1.1  78.0   71
2     1    2 1.2  76.4   72
3     1    3 1.3  77.2   73
4     2    1 2.1  68.7   68
5     2    2 2.2  67.6   67
6     2    3 2.3  68.3   68
```

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16/ 71

## Graphical functions

- ▶ BA.plot Makes a Bland-Altman plot of two methods from a data frame with method comparison data, and computes limits of agreement. The plotting etc is really done by a call to
- ▶ BlandAltman Draws a Bland-Altman plot and computes limits of agreement.
- ▶ plot.meth Plots all methods against all other, both as a scatter plot and as a Bland-Altman plot.
- ▶ bothlines Adds regression lines of  $y$  on  $x$  and vice versa to a scatter plot.

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20/ 71

## Analyses/plots in this course

- ▶ Scatter plots.
- ▶ Bland-Altman plots ( $y - x$  vs.  $(x + y)/2$ )
- ▶ Limits of agreement.
- ▶ Models with constant bias.
- ▶ Models with linear bias.
- ▶ Conversion formulae between methods (single replicates)
- ▶ Plots of conversion equations.
- ▶ Graphical reporting of variance components.

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17/ 71

## Data manipulating functions

- ▶ make.repl Generates a repl column in a data frame with columns meth, item and y.
- ▶ perm.repl Randomly permutes replicates within (method,item) and assigns new replicate numbers.
- ▶ to.wide Transforms a data frame in the long form to the wide form.
- ▶ to.long Reverses the result of to.wide.
- ▶ tab.repl Tabulates replicates by methods and items.
- ▶ sim.meth Simulates a dataset from a method comparison experiment for given parameters for bias, exchangeability and variances.

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21/ 71

## Analysis functions

- ▶ Deming Performs Deming regression, i.e. regression with errors in both variables.
- ▶ `BA.est` Estimates in the variance components models underlying the concept of limits of agreement, and returns the bias and the variance components. Assumes constant bias between methods.
- ▶ `MethComp` Estimates via BUGS in the general model with non-constant bias (and in the future) possibly non-constant standard deviations of the variance components. Produces a `MethComp` object.

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22/ 71

## Reporting functions

These functions all take a `MethComp` object as input.

- ▶ `print.MethComp` Prints a table of conversion equation between methods analyzed, with prediction standard deviations. Also gives summaries of the posteriors for the parameters that constitute the conversion algorithms.
- ▶ `plot.MethComp` Plots the conversion lines between methods with prediction limits.
- ▶ `plot.VarComp` Plots smoothed posterior densities for the variance component estimates.

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23/ 71

## Does it work?

You should get something reasonable out of this:

```
library(MethComp)
data(ox)
plot.meth(ox)
plot.meth(perm.repl(ox))
BA.plot(ox)
BA.est(ox)
BA.est(perm.repl(ox))
MethComp(ox, code.only=TRUE)
m1 <- MethComp(ox)
print(m1)
plot(m1)
plot.VarComp(m1)
```

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24/ 71

## Repeatability and reproducibility

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(Repro)

## Accuracy of a measurement method

- ▶ Repeatability:  
The accuracy of the method under exactly similar circumstances; i.e. the same lab, the same technician, and the same day.  
(**Repeatability** conditions)
- ▶ Reproducibility:  
The accuracy of the method under comparable circumstances, i.e. the same machinery, the same kit, but possibly different days or laboratories or technicians.  
(**Reproducibility** conditions)

Repeatability and reproducibility

25/ 71

## Quantification of accuracy

- ▶ Upper limit of a 95% confidence interval for the difference between two measurements.
- ▶ Suppose the variance of the measurement is  $\sigma^2$ :

$$\text{var}(y_{mi1} - y_{mi2}) = 2\sigma^2$$

i.e. the standard error is  $\sqrt{2}\sigma$ , and a confidence interval for the difference:

$$0 \pm 1.96 \times \sqrt{2}\sigma = 0 \pm 2.772\sigma \approx 2.8\sigma$$

- ▶ This is called the reproducibility coefficient or simply the reproducibility. (The number 2.8 is used as a convenient approximation).

Repeatability and reproducibility

26/ 71

## Quantification of accuracy

- ▶ Where do we get the  $\sigma$ ?
- ▶ Repeat measurements on the same item (or even better) several items.
- ▶ The conditions under which the repeat (replicate) measurements are taken determines whether we are estimating repeatability or reproducibility.
- ▶ In larger experiments we must consider the **exchangeability** of the replicates — i.e. which replicates are done under (exactly) similar conditions and which are not.

Repeatability and reproducibility

27/ 71

## Comparing two methods with replicate measurements

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(Comp-repl)

## Extension of the model: replicate measurements

$$y_{mir} = \alpha_m + \mu_i + c_{mi} + e_{mir}$$

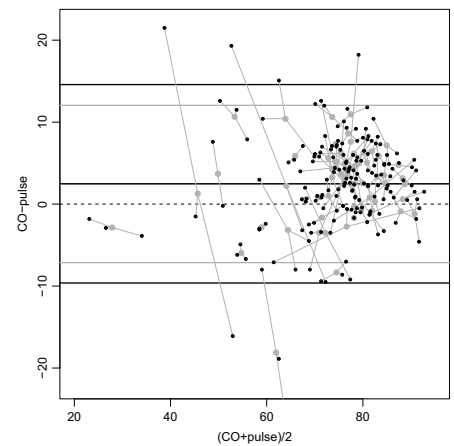
s.d. ( $c_{mi}$ ) =  $\tau_m$  — “matrix”-effect  
s.d. ( $e_{mir}$ ) =  $\sigma_m$  — measurement error

- ▶ Replicates within  $(m, i)$  is needed to separate  $\tau$  and  $\sigma$ .
- ▶ Even with replicates, the  $\tau$ s are only estimable if  $M > 2$ .
- ▶ Still assumes that the difference between methods is constant.
- ▶ Assumes *exchangeability* of replicates.

Comparing two methods with replicate measurements

28/ 71

## Oximetry data



Comparing two methods with replicate measurements

32/ 71

## Extension of the model: replicate measurements

$$y_{mir} = \alpha_m + \mu_i + a_{ir} + c_{mi} + e_{mir}$$

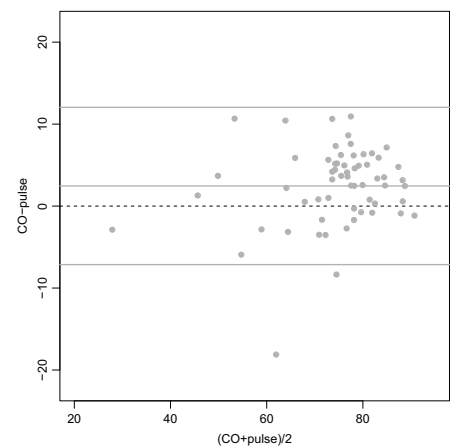
s.d. ( $a_{ir}$ ) =  $\omega$  — between replicates  
s.d. ( $c_{mi}$ ) =  $\tau_m$  — “matrix”-effect  
s.d. ( $e_{mir}$ ) =  $\sigma_m$  — measurement error

- ▶ Still assumes that the difference between methods is constant.
- ▶ Replicates are *linked* between methods:  $a_{ir}$  is common across methods, i.e. the first replicate on a person is made under similar conditions for all methods (i.e. at a specific day or the like).

Comparing two methods with replicate measurements

29/ 71

## Oximetry data



Comparing two methods with replicate measurements

33/ 71

## Replicate measurements

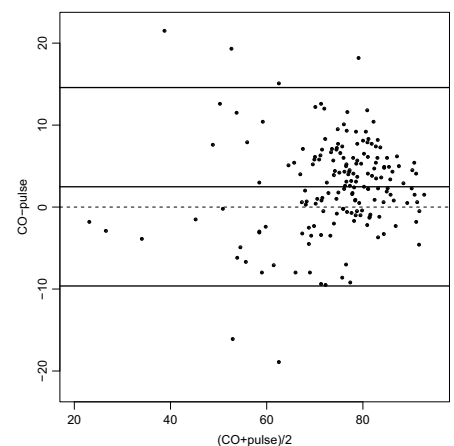
Two approaches to limits of agreement with replicate measurements:

1. Take means over replicates within each method by item stratum.
2. Replicates within item are taken as items.

Comparing two methods with replicate measurements

30/ 71

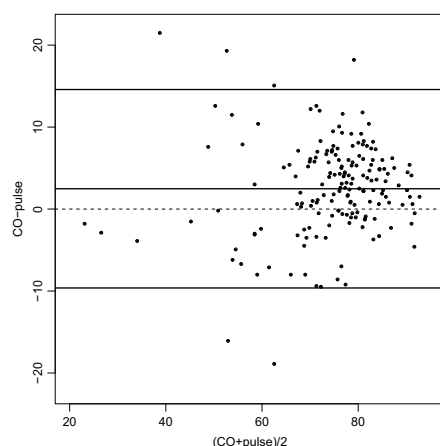
## Oximetry data



Comparing two methods with replicate measurements

34/ 71

## Oximetry data



Comparing two methods with replicate measurements

31/ 71

## Replicate measurements

- ▶ The limits of agreement should still be for difference between future **single** measurements.
- ▶ Analysis based on the **means** of replicates is therefore **wrong**:
- ▶ Model:

$$y_{mir} = \alpha_m + \mu_i + a_{ir} + c_{mi} + e_{mir}$$

- ▶  $\text{var}(y_{1jr} - y_{2jr}) = \tau_1^2 + \tau_2^2 + \sigma_1^2 + \sigma_2^2$   
— note that the term  $a_{ir} - a_{ir}$  cancels because we are referring to the *same* replicate.

Comparing two methods with replicate measurements

35/ 71

## Recommendations

- Fit the correct model, and get the estimates from that, e.g. by using BA.est.
- If you must:
  - Use linked replicates as item.
  - If replicates are not linked; make a random linking. Note: If this give a substantially different picture than using the original replicate numbering as linking key, there might be something fishy about the data.

Comparing two methods with replicate measurements

36/ 71

## $(m, r)$ - between replicates within method

This effect has  $M \times R$  levels, usually a rather small number.

This effect will therefore normally be modelled as a fixed effect, but not necessarily with  $M \times R$  parameters, presumably fewer.

If replicates are times of sampling or analysis, we may consider different time trends for each method, e.g.

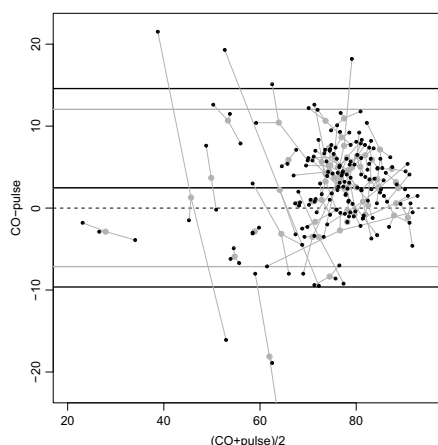
$$d_{mr} = \gamma_m t_r$$

A random  $m \times r$ -effect would be hard to interpret.

A general model

39/ 71

## Oximetry data



Comparing two methods with replicate measurements

37/ 71

## $(i, r)$ - between replicates within individual

Observations with same  $(i, r)$  — but different method — will be correlated.

Use if all methods are applied to each item at

- different times
- at different locations
- at different conditions

This means there is a minimal structure to replicates — they are linked.

There might be further structure, e.g. a systematic effect of a time.

A general model

40/ 71

## A general model

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(General)

## $(m, i)$ - between methods within individual

This is what is often called a “matrix” effect.

Matrix in the chemical sense: The surrounding matter (“matrix”) in which the stuff of interest is dissolved.

Represents random effects of items reacting differently on each measurement method.

Logical to require that the variance of these methods was allowed to differ between methods.

A general model

41/ 71

## Extension of the model:

$$\begin{aligned} y_{mir} &= \alpha_m + \mu_i + a_{ir} + c_{mi} + d_{mr} + e_{mir} \\ \text{s.d.}(a_{ir}) &= \omega \quad \text{— between replicates} \\ \text{s.d.}(c_{mi}) &= \tau_m \quad \text{— “matrix”-effect} \\ \text{s.d.}(d_{mr}) &= \nu_m \quad \text{— } m \times r \\ \text{s.d.}(e_{mir}) &= \sigma_m \quad \text{— measurement error} \end{aligned}$$

Method, Item, Replicate

- 1 3-way interaction
- 3 2-way interactions

What part of the interactions should be systematic (fixed) and what part should be random?

A general model

38/ 71

## Variance component model!

$$\begin{aligned} y_{mir} &= \alpha_m + \mu_i + a_{ir} + c_{mi} + e_{mir} \\ \text{s.d.}(a_{ir}) &= \omega \quad \text{— between replicates} \\ \text{s.d.}(c_{mi}) &= \tau_m \quad \text{— “matrix”-effect} \\ \text{s.d.}(e_{mir}) &= \sigma_m \quad \text{— measurement error} \end{aligned}$$

Note we do not consider the method by replicate interaction any more.

The model is a (standard) variance component model, where two of the variance components depend on method.

A general model

42/ 71

## Fitting the variance component model

Complicated and counter-intuitive in R:

```
> library( nlme )
> lme( y ~ meth + item,
      random = list( item = pdIdent(~meth - 1),
                    repl = ~1),
      weights = varIdent(form = ~1 | meth),
      data = ox)
```

A general model

43/ 71

```
Random effects:
Formula: ~meth - 1 | item
Structure: Multiple of an Identity
           methCO methpulse
StdDev: 2.928042 2.928042

Formula: ~1 | repl %in% item
(Intercept) Residual
StdDev: 3.415692 2.224868

Variance function:
Structure: Different standard deviations per stratum
Formula: ~1 | meth
Parameter estimates:
CO pulse
1.000000 1.795365
Number of Observations: 354
Number of Groups:
item repl %in% item
61 177
```

A general model

44/ 71

## Tease out variances for later use?

Even worse.

Therefore it has been packaged in a function that calls `lme` and then tease out the relevant parameters.

```
> BA.est(ox)
$bias
      CO      pulse
0.000000 -2.470446

$sds
MxI.CO MxI.pulse IxR resid.CO resid.pulse
2.928042 2.928042 3.415692 2.224868 3.994451

Warning message:
In pt(q, df, lower.tail, log.p) : NaNs produced
```

A general model

45/ 71

## Unequal bias

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(Lin-bias)

## Extension with non-constant bias

$$y_{mir} = \alpha_m + \beta_m \mu_i + \text{random effects}$$

There is now a *scaling* between the methods.

Methods do not measure on the same scale — the relative scaling is *estimated*, between method 1 and 2 the scale is  $\beta_2/\beta_1$ .

Consequence: Multiplication of all measurements on one method by a fixed number does not change results of analysis:

The corresponding  $\beta$  is multiplied by the same factor as is the variance components for this method.

Unequal bias

46/ 71

## Variance components

Two-way interactions:

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir}) + c_{mi} + e_{mir}$$

The random effects  $c_{mi}$ ,  $d_{mr}$  and  $e_{mir}$  have variances specific for each method.

But  $a_{ir}$  does not depend on  $m$  — must be scaled to each of the methods by the corresponding  $\beta$ .

Implies that  $\omega = \text{s.d.}(a_{ir})$  is irrelevant — the scale is arbitrary. The relevant quantities are  $\beta_m \omega$  — the between replicate variation within item *as measured on the  $m$ th scale*.

Unequal bias

47/ 71

## Variance components

**Method, Item, Replicate.**

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir}) + c_{mi} + e_{mir}$$

s.d.( $c_{mi}$ ) =  $\tau_m$

**Matrix-effect:** Each item reacts differently to each method.

If only two methods compared:

$\tau_1$  and  $\tau_2$  cannot be separated:

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

s.d.( $c_{mi}$ ) =  $\tau$

Unequal bias

48/ 71

## Variance components

**Method, Item, Replicate.**

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir}) + c_{mi} + e_{mir}$$

s.d.( $a_{ir}$ ) =  $\omega$

Common across methods — must be scaled relative to the methods.

Included if replicates are linked across methods, e.g. if there is a sequence in the replicates.

The relevant quantities to reports are  $\beta_m \omega$  — the s.d. on the scale of the  $m$ th method.

Unequal bias

49/ 71



# Conversion between methods

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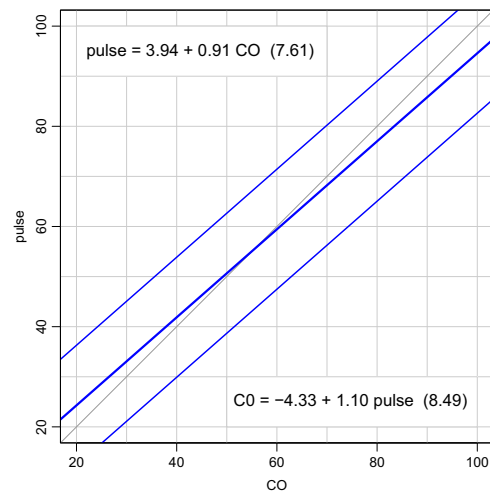
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(Convert)



Conversion between methods

53/ 71

## Predicting method 2 from method 1

$$\begin{aligned} y_{10r} &= \alpha_1 + \beta_1(\mu_0 + a_{0r}) + c_{10} + e_{10r} \\ y_{20r} &= \alpha_2 + \beta_2(\mu_0 + a_{0r}) + c_{20} + e_{20r} \\ &\Downarrow \\ y_{20r} &= \alpha_2 + \frac{\beta_2}{\beta_1}(y_{10r} - \alpha_1 - c_{10} - e_{10r}) \\ &\quad + c_{20} + e_{20r} \end{aligned}$$

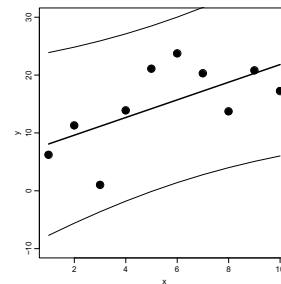
The random effects have expectation 0, so:

$$E(y_{20r}|y_{10r}) = \hat{y}_{20r} = \alpha_2 + \frac{\beta_2}{\beta_1}(y_{10r} - \alpha_1)$$

Conversion between methods

50/ 71

## What happened to the curvature?



Usually the prediction limits are curved:

$$\hat{y}|x \pm 1.96 \times \hat{\sigma} \sqrt{1 + x'x}$$

In our prediction we have ignored the last term ( $x'x$ ), i.e. effectively assuming that there is no estimation error on  $\alpha_{2.1}$  and  $\beta_{2.1}$ .

Conversion between methods

54/ 71

$$y_{20r} = \alpha_2 + \frac{\beta_2}{\beta_1}(y_{10r} - \alpha_1 - c_{10} - e_{10r}) + c_{20} + e_{20r}$$

$$\text{var}(\hat{y}_{20r}|y_{10r}) = \left(\frac{\beta_2}{\beta_1}\right)^2(\tau_1^2 + \sigma_1^2) + (\tau_2^2 + \sigma_2^2)$$

The slope of the prediction line from method 1 to method 2 is  $\beta_2/\beta_1$ .

The width of the prediction interval is:

$$2 \times 1.96 \times \sqrt{\left(\frac{\beta_2}{\beta_1}\right)^2(\tau_1^2 + \sigma_1^2) + (\tau_2^2 + \sigma_2^2)}$$

Conversion between methods

51/ 71

If we do the prediction the other way round ( $y_1|y_2$ ) we get the same relationship i.e. a line with the inverse slope,  $\beta_1/\beta_2$ .

The width of the prediction interval in this direction is:

$$\begin{aligned} &2 \times 1.96 \times \sqrt{(\tau_1^2 + \sigma_1^2) + \left(\frac{\beta_1}{\beta_2}\right)^2(\tau_2^2 + \sigma_2^2)} \\ &= 2 \times 1.96 \times \frac{\beta_1}{\beta_2} \sqrt{\left(\frac{\beta_2}{\beta_1}\right)^2(\tau_1^2 + \sigma_1^2) + (\tau_2^2 + \sigma_2^2)} \end{aligned}$$

i.e. if we draw the prediction limits as straight lines they can be used both ways.

Conversion between methods

52/ 71

# Variance components

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(Var-comp)

## Variance components

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir}) + c_{mi} + e_{mir}$$

3 variance components / random effects:

- ▶  $a_{ir}$ : between replicates within item,  $\omega^2$   
 $\beta_m \omega$  is the relevant quantity.
- ▶  $c_{mi}$ : matrix effect  $\tau_m^2$   
 $\tau_m$  is the relevant quantity.
- ▶  $e_{mir}$ : measurement error, residual variation  $\sigma_m^2$   
 $\sigma_m$  is the relevant quantity.

Variance components

55/ 71



## Variance components

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir}) + c_{mi} + e_{mir}$$

The total variance of a measurement is:

$$\sqrt{\beta_m^2 \omega^2 + \tau_m^2 + \sigma_m^2}$$

These are the variance components reported by `print.MethComp` and shown by `plot.VarComp`.

Variance components

56/ 71

## Repeatability and reproducibility

Repeatability is based on the difference between measurements made under comparable, though not exactly identical conditions.

Reproducibility is based on the difference between measurements made under comparable, though not exactly identical conditions.

This is a different setting from the one underlying the modelling of data from a comparison experiment.

The exchangeability has no meaning, we are discussing future measurements in different circumstances.

Variance components

57/ 71

## Repeatability and reproducibility

Repeatability:  $2.8\sigma_m$ :  
same individual, same replicate, but not considering the variation that constitute differences between replicates *in the experiment*.

Hence *reproducibility* is not estimable from a classical experiment, unless an extra layer of replication is introduced — i.e. different laboratories.

Variance components

58/ 71

## Implementation in BUGS

Friday 15 February

**Bendix Carstensen**

Analysis of Method Comparison Studies

15 February 2008

MEGA Center, SPH, University of Melbourne

(BUGS-impl)

## Implementation in BUGS

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

Non-linear hierarchical model:

Implement in BUGS.

- ▶ The model is *symmetrical* in methods.
- ▶ Mean is overparametrized.
- ▶ Choose a prior (and hence posterior!) for the  $\mu_s$  with finite support.
- ▶ Keeps the chains nicely in place.

Implementation in BUGS

59/ 71

## Results from fitting the model

The posterior dist'n of  $(\alpha_m, \beta_m, \mu_i)$  is singular.

But the relevant translation quantities are identifiable:

$$\alpha_{2.1} = \alpha_2 - \alpha_1\beta_2/\beta_1$$

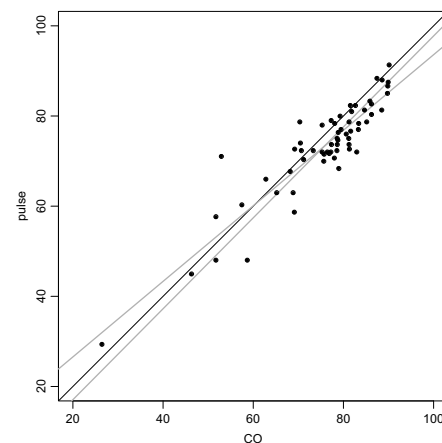
$$\beta_{2.1} = \beta_2/\beta_1$$

So are the variance components.

Posterior medians used to devise prediction equations with limits.

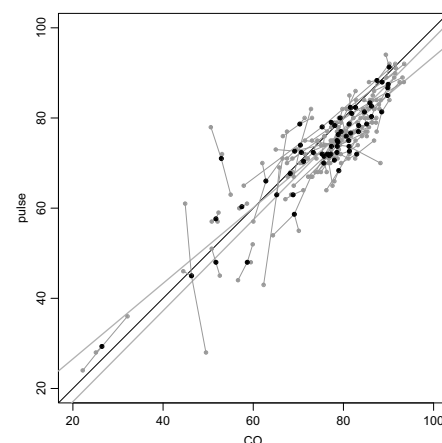
Implementation in BUGS

60/ 71



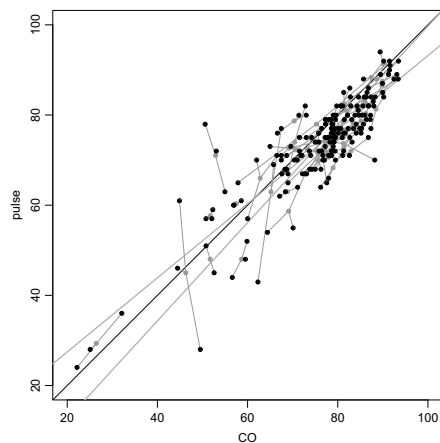
Implementation in BUGS

60/ 71



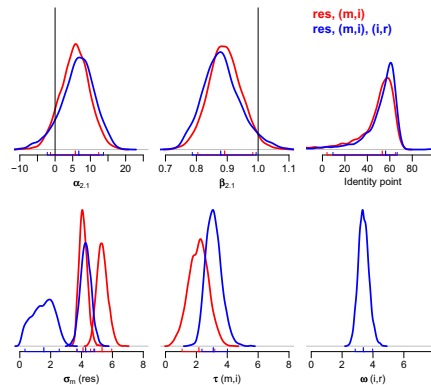
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60/ 71



Implementation in BUGS

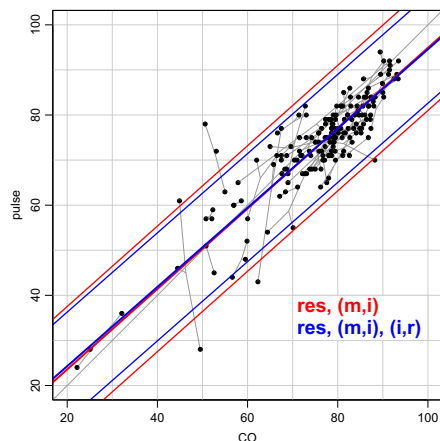
60/ 71



$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

Implementation in BUGS

61/ 71



$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

Implementation in BUGS

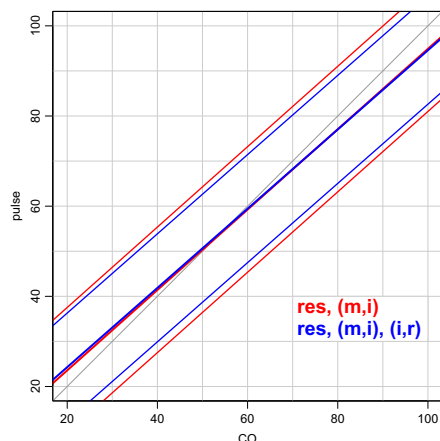
60/ 71

## Morale

- Use a proper model for your problem.
- Get the exchangeability right.
- Report the model in a useful way.

Implementation in BUGS

62/ 71



$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

Implementation in BUGS

60/ 71

## The MethComp package for R

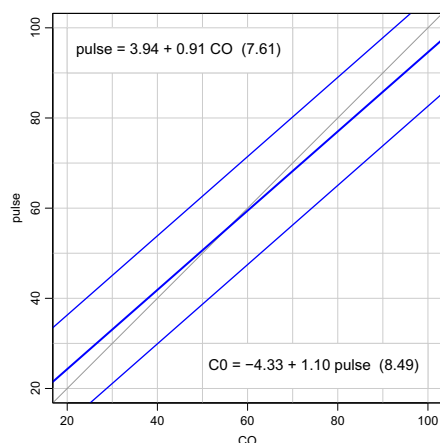
Implemented model:

$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

- Replicates required.
- R2WinBUGS is required.
- Dataframe with variables meth, item, repl and y.
- The function MethComp writes a BUGS-program, initial values and data to files.
- Runs WinBUGS and sucks results back in to R, and gives a nice overview of the conversion equations.

Implementation in BUGS

63/ 71



$$y_{mir} = \alpha_m + \beta_m(\mu_i + a_{ir} + c_{mi}) + e_{mir}$$

Implementation in BUGS

60/ 71

## Example output: Oximetry

```
> ox.mi.ir <- MethComp( ox, n.iter=5000 )
> ox.mi.ri
```

Comparison of 2 methods, using 354 measurements on 61 individuals, with up to 3 replicate measurements. ( 2 \* 61 \* 3 = 366 ):

No. individuals with measurements on each method:

	# replicates
Method	1 2 3 Sum
CO	1 4 56 61
pulse	1 4 56 61

Implementation in BUGS

64/ 71

## Example output: Oximetry

Conversion formulae ( $y_{to} = \alpha + \beta y_{from} \pm 2 \cdot sd.pred$ ):

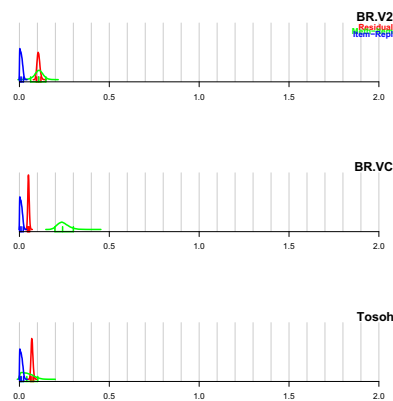
	From: CO			pulse		
To:	alpha	beta	sd.pred	alpha	beta	sd.pred
CO	0.000	1.000	4.266	-4.328	1.098	8.487
pulse	3.939	0.911	7.606	0.000	1.000	5.534

Variance components (standard deviations):

	50%	2.5%	97.5%	0%	100%
sigma.mir[CO]	1.6285	0.2092	2.8274	0.0724	3.4330
sigma.mir[pulse]	4.2580	3.5390	4.9725	3.0670	5.9800
sigma.mi[CO]	4.8043	2.7504	13.3685	2.2597	17.6134
sigma.mi[pulse]	4.3123	2.4981	11.5859	1.9248	13.2186
sigma.ir[CO]	3.9213	3.1452	4.7038	2.7289	5.3129
sigma.ir[pulse]	3.5433	2.7542	4.3516	2.2610	4.8723

Implementation in BUGS

65/ 71



Implementation in BUGS

69/ 71

## HbA<sub>1c</sub> - 3 different instruments

```
> hbv.mi.ir <- MethComp(hbv, n.iter=5000)
> print(hbv.mi.ir, across=FALSE)
```

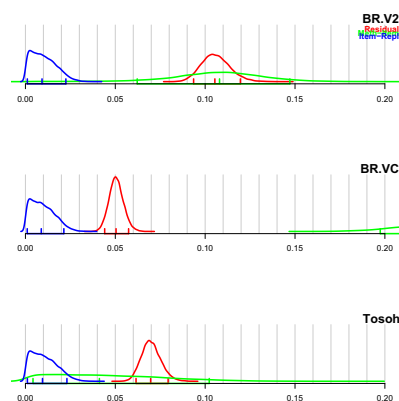
Conversion formula:

$y_{to} = \alpha + \beta y_{from} \pm 2 \cdot sd.pred$

	From:	BR.V2	BR.VC	Tosoh
To:				
BR.V2	alpha	0.000	-1.627	1.413
	beta	1.000	1.154	0.946
	sd.pred	0.254	2.079	2.099
BR.VC	alpha	1.417	0.000	2.412
	beta	0.867	1.000	0.819
	sd.pred	1.800	0.164	1.927
Tosoh	alpha	-1.591	-3.144	0.000
	beta	1.057	1.220	1.000
	sd.pred	2.145	2.249	0.156

Implementation in BUGS

66/ 71



Implementation in BUGS

70/ 71

## HbA<sub>1c</sub> - 3 different instruments

Variance components (standard deviations):

	50%	2.5%	97.5%	0%	100%
sigma.mir[BR.V2]	0.2089	0.1816	0.2401	0.1614	0.2692
sigma.mir[BR.VC]	0.1074	0.0813	0.1286	0.0642	0.1467
sigma.mir[Tosoh]	0.0345	0.0006	0.0824	0.0004	0.0984
sigma.mi[BR.V2]	1.3495	1.0780	1.7742	0.9194	2.1615
sigma.mi[BR.VC]	1.3088	1.0498	1.6979	0.8615	2.1350
sigma.mi[Tosoh]	1.4416	1.0782	5.3653	0.9250	6.3534
sigma.ir[BR.V2]	0.1418	0.1037	0.1882	0.0855	0.2319
sigma.ir[BR.VC]	0.1239	0.0928	0.1572	0.0797	0.1827
sigma.ir[Tosoh]	0.1496	0.1231	0.1815	0.0950	0.2002

Implementation in BUGS

67/ 71

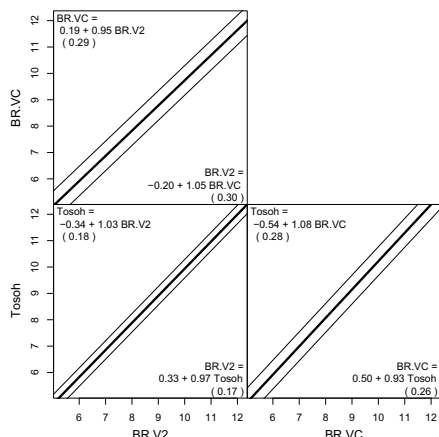
## The MethComp package

Also (currently) contains:

- ▶ `BA.plot` — make a Bland-Altman plot and compute limits of agreement.
  - ▶ `BA.est` — estimates in the variance component model for the constant bias situation.
  - ▶ `Deming` — regression with errors in both variables.
- A .pdf with a detailed derivation of the formulae (by Anders C Jensen) is included in the package too.
- ▶ A number of example data sets, amongst them all examples from [?].

Implementation in BUGS

71/ 71



Implementation in BUGS

68/ 71